

NOTES ON COBORDISM THEORY
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A Detailed Table of Contents
 compiled by Peter Landweber and Doug Ravenel in November, 2007
 based on decades of careful reading

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