Workshop
Harmonic Analysis and the Renormalization Group
University of Virginia, April 18–21, 2014

TITLES AND ABSTRACTS

Abdelmalek Abdesselam (University of Virginia)
Title: Introduction to rigorous renormalization group theory.
Abstract: In these lectures we will present an introduction to the renormalization group (RG) that assumes no prior knowledge of the subject. We will show how the RG strategy provides a roadmap for the construction of functional integrals as they apprear in modern quantum field theory. The first part will be essentially conjectural with emphasis on intuition and the identification of the main mathematical obstacles. In the second part we will study a simplified hierarchical model for which these obstacles have been overcome and we will explain the methods that allow one to do that. Finally in the last part we will present some of the tools, in need of improvement (perhaps by new input from harmonic analysis), which apply to the more realistic Euclidean model such as the polymer representation and multiscale cluster expansions in phase space.

Fabrice Baudoin (Purdue University)
Title: An introduction to rough paths theory.
Abstract: In this short course we will give a self-contained introduction to the theory of rough paths that was developed by T. Lyons in the 1990's to understand differential equations driven by very irregular signals. When applied to Brownian motion the rough paths theory allows to recover Stratonovitch stochastic differential equations, but it also allows to deal with much rougher signals like the fractional Brownian motion with low parameter.

Yen Do (Yale University)
Title: Introduction to phase plane analysis.
Abstract: Phase plane analysis is about decomposing signals into pieces that are essentially localized in time and frequency. Analysis of these pieces and their interactions play key roles in proving various deep theorems about Fourier series and multilinear operators. These talks will be an introduction

to this subject. In the first talk, we will focus on a discrete setting with simpler geometric properties. In the second talk, the more delicate continuous setting will be discussed.

Zoran Grujic (University of Virginia)
Title: From super-criticality to log sub-criticality of the 3D Navier-Stokes problem: a roadmap.
Abstract: The goal of this lecture is to present – in a somewhat self-contained manner – several recent results revealing an experimentally, numerically, and mathematical analysis-supported geometric scenario manifesting large data logarithmic sub-criticality of the 3D Navier-Stokes regularity problem. Shortly, in this scenario, the transversal small scales produced by the mechanism of vortex stretching (coupled with the decay of the volume of the regions of intense vorticity) reach the threshold sufficient for the locally anisotropic diffusion to engage and control the sup-norm of the vorticity, preventing the formation of (possible) finite time singularities. The tools utilized include geometric function theory, a bit of geometric measure theory, some statistical mechanics-like constructs, and harmonic analysis.

Martin Hairer (University of Warwick)
Title: Introduction to regularity structures.
Abstract: The construction of Euclidean Φ^4 field theory in three (space-time) dimensions was one of the main achievements of the programme of constructive QFT. A natural interpretation of the probability measure built in this way is that it should describe the mesoscopic fluctuations of the order parameter for systems in the Ising universality class at equilibrium near their phase transition and in near mean-field regimes. In this context, it is natural to look for evolutions that are local and keep Φ^4_3 invariant in order to describe the dynamical fluctuations of the order parameter near the phase transition. These evolutions should be either of Glauber type (with no additional conserved quantity) or of Kawasaki type (keeping the total "magnetisation" fixed). The goal of these lectures is to show how such dynamics can be built and to highlight some of the properties of the construction. In turn, this sheds new light on the universality of the various approaches to the construction of Φ^4_3.

Camil Muscalu (Cornell University)
Title: Iterated Fourier series and the physical world.
Abstract: At first, we plan to describe a very interesting (and natural) physical reality problem, whose study relies on the almost everywhere convergence of iterated Fourier series.

Then, we would like to relate the simplest of them, with other well known objects of harmonic analysis, such as Carleson's maximal operator and Calderón commutators. Ideally, all of these should take at most half of the time of my lectures.

The rest of the time, will be spent on more technical, related issues, in a way that complements Yen Do's lectures. For instance, one possibility would be to describe in detail the one scale cases of paraproducts, bilinear Hilbert transform and Carleson's operator.

Michael O'Carroll (ICMC-USP Sao Carlos)
Title: Stability and multi-body interactions.
Abstract: We consider the quantum field theory associated with some statistical mechanics models via the Feynman-Kac formula and the Osterwalder-Schrader construction. The construction provides a positive, self-adjoint energy operator H and self-adjoint spatial momentum component operators. To obtain detailed spectral content of these operators additional information about correlation functions is needed, i.e. the behavior of their convolution inverses. Spectral subsets of H are associated with N particle Hamiltonians. The positiveness of their spectrum is due to the presence of multi-particle potentials (more than pair potentials) and is essential for the positivity of H.

Hao Shen (Princeton University)
Title: Renormalization group by harmonic extensions.
Abstract: We discuss a new renormalization group method based on conditional expectations and harmonic extensions, to study functional integrals associated with small gradient perturbations of Gaussian fields. In this new method one implements RG by solving elliptic PDEs in local domains at all the scales. We apply this method to the model of classical dipole gas on the lattice. In the end of the talk, I will propose some possibilities or challenges of applying this method to other related models, where harmonic analysis may shed some light.